Synthesis of Planar Microwave Band-pass Filter
based on Foster-type Network and Normal Mode Expansion Method

Hsu,Jui Pang T.Anada H.Tsubaki Y.Migita K.Nagashima
Faculty of Engineering, KANAGAWA UNIVERSITY

Abstract **A new synthesis method of microwave filter circuit based on the Foster-type network representation is proposed, where two port impedance matrix which realizes the desired frequency characteristics and that of any microwave circuit structure are expanded into Foster-type network representation; microwave filter circuit can be synthesized by matching the both network representation. In this paper fundamental idea of synthesis method and practical examples are explained.**

I. Introduction

Now systematic method of microwave filter synthesis suitable for CAD is strongly needed with the recent wide-spread commercial use of microwaves and development of new materials and structures. It is well-known that there are two typical starting network representation for the synthesis of microwave filter circuit, i.e., Cauer(ladder)-type and Foster (Resonator)-type(1) as shown in fig.1. So far the former has been frequently used, but the latter has not because the ideal transformer is difficult to be realized. However the former is also encountering serious problems at microwave frequency such as unwanted frequency response of the constitutive circuit elements, and excitation of field disturbance and their mutual interaction at discontinuities of circuit element connection, which often removes the frequency characteristics far away from the desired one. It seems not impossible but complicated to establish general and systematic way which takes into account these problems for exact synthesis.

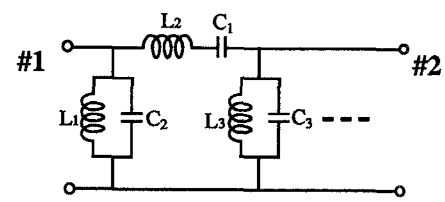
In order to overcome these problems, we propose to use Foster-type network representation. This is because normal mode expansion method(2) always gives the Foster-type equivalent network representation for any microwave circuit structure. This also means that ideal transformer is realizable. The advantages of using this network representation are

1. Problems of unwanted frequency response of circuit elements and discontinuities are naturally absorbed into practical equivalent network parameters through normal mode calculation.
2. Resonator, fundamental constitutive circuit element, is easily obtainable at microwave frequency.

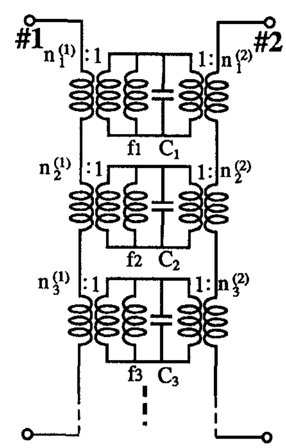
In the following, fundamental idea of how to synthesize microwave filter circuit based on Foster-type network representation and normal mode expansion method are explained. Then as an example of the synthesis, maximally-flat band-pass filters are synthesized in two dimensional planar structure(3) of stub-type. Finally the corresponding strip-line filters are fabricated and measured. The results agree well with the theory, which demonstrates the validity of the synthesis mentioned here.

II. Fundamental Idea of Synthesis Method

Fundamental idea of how to synthesize microwave filter circuit is summarized in fig.2.



(a) Cauer (ladder) type network representation



(b) Foster(resonator) type network representation

Fig.1 Two types of network representation.

Network parameters of Foster-type to be realized(fig.b) are derived from the desired frequency characteristics(fig.a) based on the conventional circuit theory. Also equivalent network parameters (fig.c) of any microwave planar structure(fig.d) can be calculated by analytical method or computer based on the normal mode expansion method. Equivalent network parameters are, in general, function of microwave circuit dimension. Then the synthesis takes the following procedure.

1. Normal modes of planar circuit are properly selected to match the corresponding resonator of the desired Foster-type network.
2. Microwave planar circuit dimensions are adjusted to equate each equivalent network parameter to the desired value in fig.b.
3. After equating every equivalent network parameter, microwave circuit dimensions are determined, which means the completion of synthesis.

If the microwave circuit configuration is properly chosen and number of freedom of microwave circuit dimension are limited to necessary number, these microwave planar circuit structure is uniquely determined.

1992 IEEE MTT-S Digest

Practical Example

In order to show how to synthesize the practical microwave filter based on the forementioned idea, maximally-flat band-pass filters are synthesized with stub-type planar configuration.

III. Derivation of Foster-type Network Parameter for Maximally-flat Band-pass Filter

Two port impedance matrix which realizes maximally-flat 3dB band-pass filter (ω_0 =center angular frequency, r=fractional bandwidth, Z_0 =Load impedance) can be derived from the insertion loss method in general,but is easily given by Cauer-type network parameter tables. Then the impedance is expanded in terms of poles,which gives Foster-type network parameters. The results up to 5 resonators are shown in table2. Ideal transformer ratio of n-th resonator $n_n^{(i)}$ is given by eq.(1) in terms of Q_n, which is also given in table 2.

$$\frac{n_n^{(i)^2}}{C_n} = \frac{\omega_n}{Q_n} \cdot Z_0 \quad (i = 1, 2) \qquad (1)$$

IV.Derivation of Equivalent Network Parameter for Planar Circuit

We suppose that filter characteristics are embedded in stub-type planar structure(3). It is well known that mode impedance of an arbitrary shaped two port planar circuit shown in fig.2(d) is given by eq.(2)(4)(5)(6) through modal analysis with proper definition of mode voltage and mode current(7).

$$Z_{p,q}^{i,j} = -j \frac{1}{C_0} \sum_{n=0}^{\infty} \frac{\omega}{\omega^2 - \omega_n^2} n_{np}^{(i)} n_{nq}^{(j)} \quad (i,j = 1, 2) \qquad (2)$$

$C_0 = \varepsilon S / d$: capacitance of planar circuit

$\omega_n^2 = k_n^2 / \varepsilon\mu$: resonant angular frequency of n-th mode

Table 1 Normal mode function

$$\frac{\partial^2 \varphi_n}{\partial x^2} + \frac{\partial^2 \varphi_n}{\partial y^2} + k_n^2 \varphi_n = 0 \quad \text{in S}$$

$$\mathbf{n} \cdot \text{grad}\, \varphi_n = 0 \quad \text{on C}$$

$$k_0 = 0,\ k_1 \leq k_2 \leq \cdots$$

$$\frac{1}{S} \iint_S \varphi_n \cdot \varphi_m \, dxdy = \delta_{nm}$$

S: Area of planar circuit

$n_{n,p}^{(i)}$ is ideal transformer ratio between n-th planar mode and p-th mode of i-th transmission line and given by eq.(3).

$$n_{n,p}^{(i)} = \frac{1}{W^{(i)}} \int_0^{W^{(i)}} \varphi_n (x, y) \cdot f_p^{(i)}(s^{(i)})\, ds^{(i)} \qquad (3)$$

where $\varphi_n (x, y)$ is normal mode function of planar circuit given in table1 and $f_p^{(i)}(s^{(i)})$ is that of p-th width mode in i-th transmission line.

$$f_p^{(i)}(s^{(i)}) = \sqrt{\varepsilon_p} \cos \frac{p\pi s^{(i)}}{W^{(i)}} \quad (p = 0, 1, 2 \ldots\ldots) \qquad (4)$$

When every width of input/output planar transmission line is narrow compared with operating wavelength, higher width mode can be neglected. Then effective dominant mode impedance is approximately given by TEM mode impedance $Z_{0,0}^{i,j}$,i.e.

$$Z_{\text{eff}}^{i,j} = -j \frac{1}{C_0} \sum_{n=0}^{\infty} \frac{\omega}{\omega^2 - \omega_n^2} n_n^{(i)} n_n^{(j)} \ , \quad n_n^{(i)} = n_{n0}^{(i)} \qquad (5)$$

Hence the corresponding equivalent network is given in fig.2(c), where $n_0^{(1)} = n_0^{(2)} = 1.0$

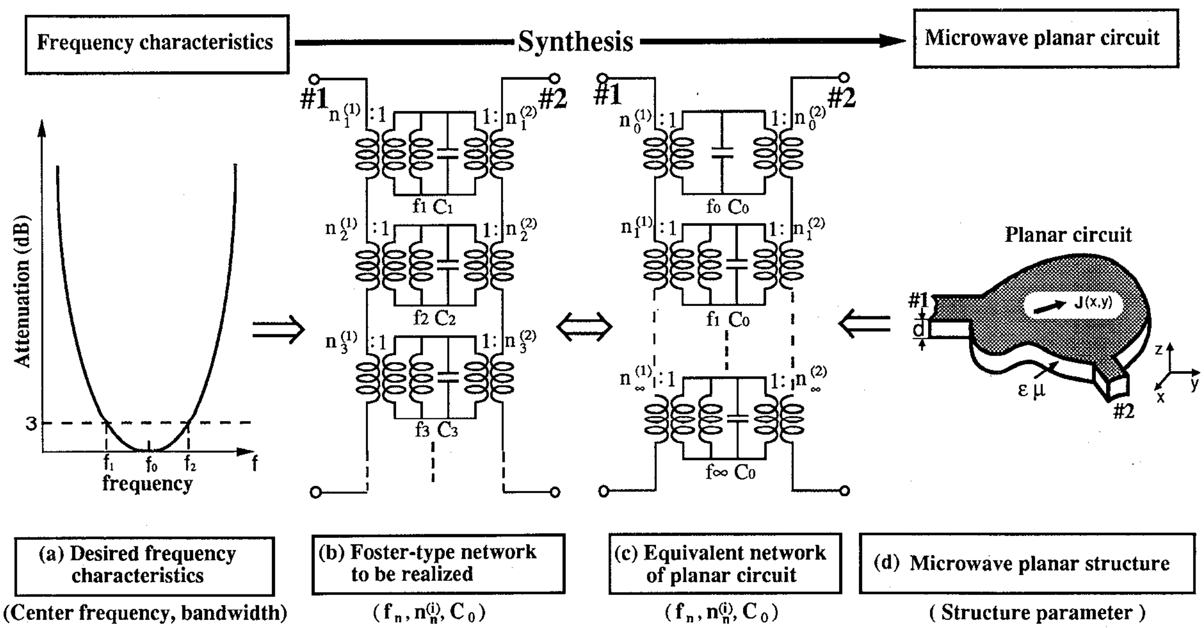


Fig.2 Scheme of synthesis method based on the Foster-type network and normal mode expansion

V. Synthesis of 3dB Maximally-flat Band-pass Filter in Stub-type Planar Circuit.

Following the scheme of Fig.2, practical stub-type planar circuit is synthesized,which realized 3dB maximally-flat band-pass filter.Many cases are synthesized where operating center frequency, fractional band-width and number of resonator are varied. Stripline structure and Rexolite 2200(ε_s=2.62) is used. Results of 3-stub cases with 20% fractional band-width and center frequency of 3 GHz are shown in fig. 3 .

In these figures one dimensional(1D) synthesized results and two dimensional(2D) synthesized results are shown in the left and right column respectively.The results are as follows

1.(a1) and (a2) show synthesized structure
2.(b1) and (b2) are realized frequency characteristics(solid line), and the desired frequency characteristics(dotted line).
3.(c1) and (c2) show the working modes (underlined) and out of band (spurious) modes.
4.Tables show the desired network parameters and realized network parameters with 2-dimensional planar case.

Also two dimensional structure translated from one dimensional synthesized result(a3), their frequency characteristics(b2) and their network parameters(table) are shown in the figures.

VI. Verification by Experiment

The practical stub-type planar circuits are fabricated in strip-line where fringing capacitance are taken into consideration.One of the experimental results (3GHz, 20%) are shown in fig.4 with theoretical results. Good agreement between theory and measurements demonstrates the validity of the synthesis method proposed here.

VII. Conclusion

New synthesis method of planar microwave band pass filter based on Foster-type network and normal mode expansion method is proposed. Maximally-flat band-pass filters are synthesized, fabricated and measured. Through the corresponding experiment, validity and usefulness of new synthesis method is shown. In future possibility of the control of stop band characteristics based on the same idea will be investigated

Reference

1.R.M.Foster, "A Reactive Theorem", B.S.T.J. vol.3, 1924, pp.259-267
2.S.A.Schlkunoff, "Representation of Impedance Function in terms of Resonant Frequencies", Proc.I.R.E.,32,83(1944)
3.T.Okoshi and T.Miyoshi, "The Planar Circuit- an Approach to Microwave Integrated Circuitry", IEEE. MTT.vol.20,no.4, pp.245-252, April, 1972.
4.P.P.Civalleri and S.Ridella, "Impedance and Admittance Matrices of Distributed Three Layer N-Ports", IEEE. CT. vol. 17, pp.392 - 398, Aug.1970.
5.Hsu, Jui-Pang, T.Anada and O. Kondo, "Analysis of Microwave Planar Circuits by Normal Mode Method", Trans. IECE' 75/12 vol.58-B, no.12, pp.671-678, Dec.1975. Elec tronics and Communications in Japan ,vol.58-B, No.12, pp.93 101, 1975 (translated)
6.G.D'Inzeo, F.Giannini and R.Sorrentino, "Wide-band Equivalent Circuits of MicrowavePlanar Networks", IEEE.MTT. vol.28, no.10,pp.1107-1113,Oct.1980.
7.Hsu, jui-Pang and T.Anada, "Planar Circuit Equation and Its Practical Application to Planar-type Transmission Line Circuit", IEEEMTT. International Symp. DIGEST pp.574- 576 (V2), 1983.

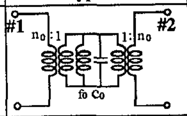
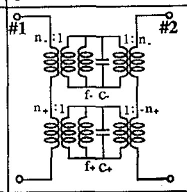
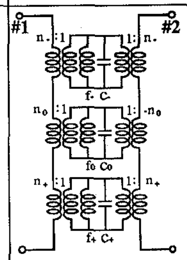
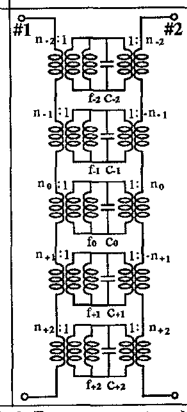
Pole	Foster-type Network	2 port impedance matrix	Circuit parameter
1		$\frac{Z}{Z_0}=-j\frac{1/Q_0}{\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}\begin{pmatrix}1&1\\1&1\end{pmatrix}$	$Q_0=\frac{2}{r}$
2		$\frac{Z}{Z_0}=-j\frac{1/Q_-}{\left(\frac{\omega}{\omega_-}-\frac{\omega_-}{\omega}\right)}\begin{pmatrix}1&1\\1&1\end{pmatrix}-j\frac{1/Q_+}{\left(\frac{\omega}{\omega_+}-\frac{\omega_+}{\omega}\right)}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$	$Q_\pm\doteqdot\frac{2\sqrt{2}}{r}\sqrt{1+\frac{r^2}{8}}$ $\frac{\omega_\pm}{\omega_0}\doteqdot\sqrt{1+\frac{r^2}{8}}\pm\frac{r}{2\sqrt{2}}$
3		$\frac{Z}{Z_0}=-j\frac{1/Q_-}{\left(\frac{\omega}{\omega_-}-\frac{\omega_-}{\omega}\right)}\begin{pmatrix}1&1\\1&1\end{pmatrix}-j\frac{1/Q_0}{\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}-j\frac{1/Q_+}{\left(\frac{\omega}{\omega_+}-\frac{\omega_+}{\omega}\right)}\begin{pmatrix}1&1\\1&1\end{pmatrix}$	$Q_0=\frac{2}{r}$ $Q_\pm=\frac{4}{r}\sqrt{1+\frac{r^2}{4}}$ $\frac{\omega_\pm}{\omega_0}=\sqrt{1+\frac{r^2}{4}}\pm\frac{r}{2}$
5		$\frac{Z}{Z_0}=-j\frac{1/Q_{-2}}{\left(\frac{\omega}{\omega_{-2}}-\frac{\omega_{-2}}{\omega}\right)}\begin{pmatrix}1&1\\1&1\end{pmatrix}-j\frac{1/Q_{-1}}{\left(\frac{\omega}{\omega_{-1}}-\frac{\omega_{-1}}{\omega}\right)}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}-j\frac{1/Q_0}{\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}\begin{pmatrix}1&1\\1&1\end{pmatrix}-j\frac{1/Q_{+1}}{\left(\frac{\omega}{\omega_{+1}}-\frac{\omega_{+1}}{\omega}\right)}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}-j\frac{1/Q_{+2}}{\left(\frac{\omega}{\omega_{+2}}-\frac{\omega_{+2}}{\omega}\right)}\begin{pmatrix}1&1\\1&1\end{pmatrix}$	$Q_0=\frac{3.236}{r}$ $Q_{\pm1}=\frac{2.472}{r}\sqrt{1+\frac{r^2}{4}}$ $Q_{\pm2}=\frac{4}{r}\sqrt{1+\frac{r^2}{2.472}}$ $\frac{\omega_{\pm1}}{\omega_0}=\sqrt{1+\frac{r^2}{4}}\pm\frac{r}{2}$ $\frac{\omega_{\pm2}}{\omega_0}=\sqrt{1+\frac{r^2}{2.472}}\pm\frac{r}{\sqrt{2.472}}$

Table 2 Foster-type network parameter for 3 dB Maximally Flat Band pass filter (ω_0:center frequency, r=fractional band width)

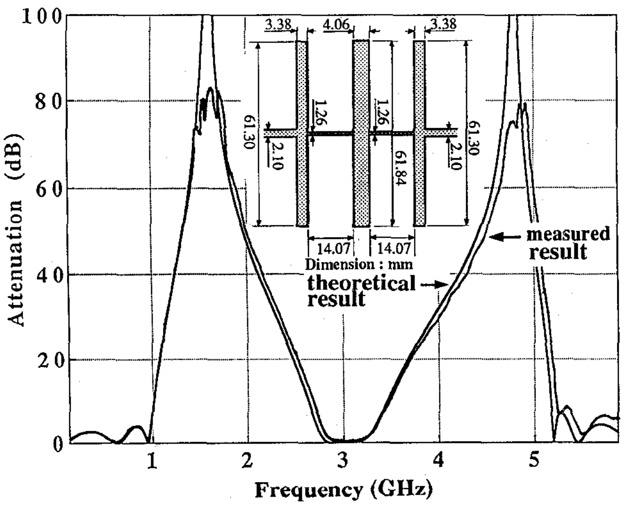
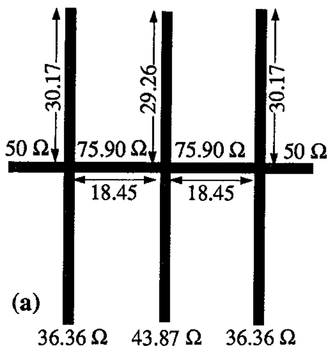
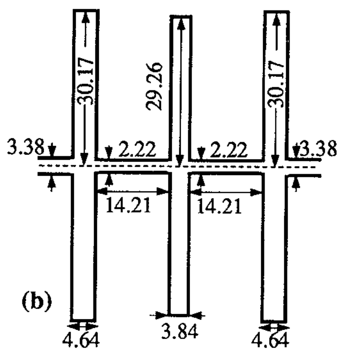


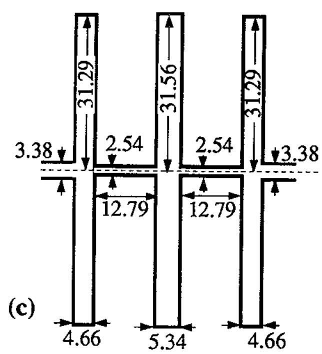
Fig.4 Measured frequency characteristics of synthesized structure with theoretical results.



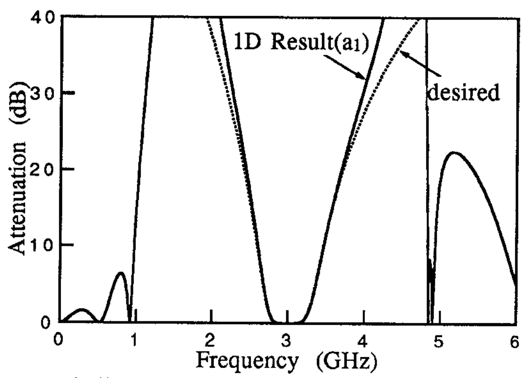
(a1) synthesized 1-D structure



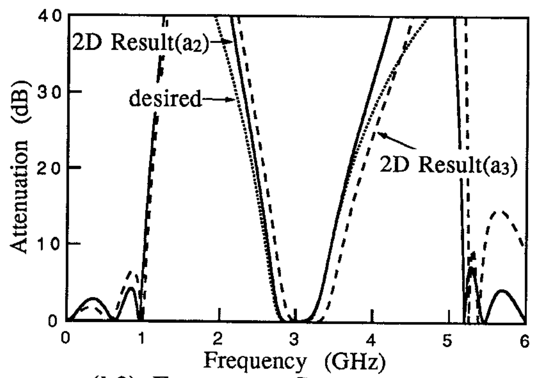
(a3) translated 2-D structure



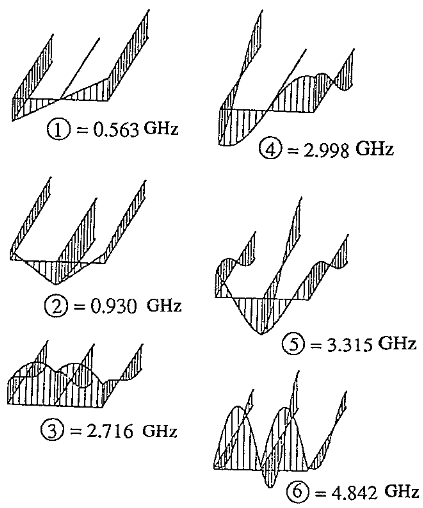
(a2) synthesized 2-D structure



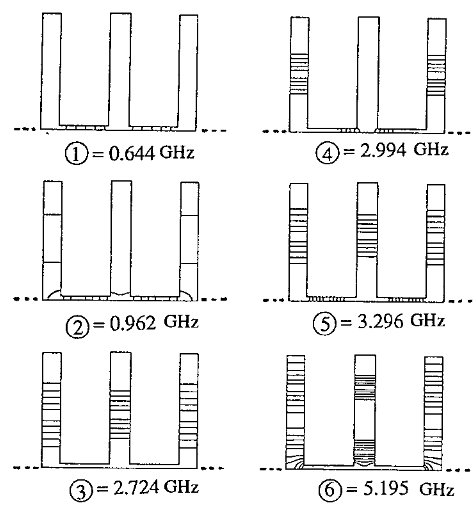
(b1) Frequency Characteristics(1D)



(b2) Frequency Characteristics(2D)



(c1) Normal Modes



(c2) Normal Modes

Network parameter	f_3	f_4	f_5	n_3^2/C_0	n_4^2/C_0	n_5^2/C_0
Desired value	2.714	3.000	3.314	0.848	1.884	1.036
Translated 2D case	2.819	3.066	3.423	0.926	1.965	0.996
Synthesized 2D case	2.724	2.994	3.296	0.833	1.919	1.015

$\varepsilon_s = 2.62$

$[n^2/C_0] = 10^{-1}[pF]^{-1}$

$[f] = [GHz]$

Fig.3 1D and 2D synthesized results with frequency characteristics and normal modes (3 stage, f_0 = 3GHz, r = 20%)